# ASSET ALLOCATION: FALLACIES, CHALLENGES, AND SOLUTIONS

WILLIAM KINLAW MARK KRITZMAN DAVID TURKINGTON

#### Fallacy: Asset allocation determines more than 90 percent of performance

Fallacy: Time diversifies risk

Fallacy: Optimized portfolios are hypersensitive to input errors

Fallacy: Factors offer superior diversification and noise reduction

## THE IMPORTANCE OF ASSET ALLOCATION

#### Illustration of BHB Methodology

Period	Stock A	Stock B	Stock Index	Bond A	Bond B	Bond Index	Skillful Manager	Unlucky Manager
1	15.0%	7.5%	11.3%	15.0%	7.5%	11.3%	15.0%	7.5%
2	8.0%	4.0%	6.0%	8.0%	4.0%	6.0%	8.0%	4.0%
3	-1.0%	-0.5%	-0.8%	-1.0%	-0.5%	-0.8%	-1.0%	-0.5%
4	-14.0%	-7.0%	-10.5%	-14.0%	-7.0%	-10.5%	-14.0%	-7.0%
5	4.0%	2.0%	3.0%	4.0%	2.0%	3.0%	4.0%	2.0%
6	32.0%	16.0%	24.0%	32.0%	16.0%	24.0%	32.0%	16.0%
7	18.0%	9.0%	13.5%	18.0%	9.0%	13.5%	18.0%	9.0%
8	6.0%	3.0%	4.5%	6.0%	3.0%	4.5%	6.0%	3.0%
9	24.0%	12.0%	18.0%	24.0%	12.0%	18.0%	24.0%	12.0%
10	8.0%	4.0%	6.0%	8.0%	4.0%	6.0%	8.0%	4.0%
Average	10.0%	5.0%	7.5%	10.0%	5.0%	7.5%	10.0%	5.0%

## THE IMPORTANCE OF ASSET ALLOCATION



Asset Class Performance

Fallacy: Asset allocation determines more than 90 percent of performance

#### Fallacy: Time diversifies risk

Fallacy: Optimized portfolios are hypersensitive to input errors

Fallacy: Factors offer superior diversification and noise reduction

It is widely assumed that investing over long horizons is less risky than investing over short horizons, because the likelihood of loss is lower over long horizons.

#### Time, Volatility, and Probability of Loss

Expected continuous return: 10% Continuous standard deviation: 20%

Investment Horizon	Annualized Continuous Standard Deviation	Probability of Loss (<0%) on Average over Horizon
1 Year	20.0%	30.9%
5 Years	8.9%	13.2%
10 Years	6.3%	5.7%
20 Years	4.5%	1.3%

Paul A. Samuelson showed that time does not diversify risk, because though the probability of loss decreases with time, the magnitude of potential losses increases with time.

Expected utility accounts for both the likelihood and magnitude of changes in wealth.

A certainty equivalent is the certain amount that conveys the same expected utility as a risky gamble.

ln(\$100) = 4.6052

50% x *ln*(\$100 x 1.3333) + 50% x *ln*(\$100 x 0.75) = 4.6052

#### Expected Wealth and Expected Utility

	Initial Wealth	1st Period Distribution	2nd Period Distribution	3rd Period Distribution
			477 70 05	237.04 x .125
		100 00 v E0	177.78 X .25	133.33 x .125
		133.33 X .50	100.00 × 25	133.33 x .125
	100.00		100.00 X .25	75.00 x .125
	100.00		100.00 × 25	133.33 x .125
		75.00 x .50	100.00 x .25	75.00 x .125
			56 25 y 25	75.00 x .125
			50.25 X .25	42.19 x .125
Expected wealth	100.00	104.17	108.51	113.03
Expected utility	4.6052	4.6052	4.6052	4.6052

It is also true that the probability of loss within an investment horizon never decreases with time.

$$Pr_W = N\left[\frac{\ln(1+L)-\mu T}{\sigma\sqrt{T}}\right] + N\left[\frac{\ln(1+L)+\mu T}{\sigma\sqrt{T}}\right] (1+L)^{\frac{2\mu}{\sigma^2}}$$

Probability of a Within-Horizon Loss

Continuous Expected Return:	10%
Continuous Standard Deviation:	20%

Investment	Probability
Horizon	of -10%
0.25 Years	22.1%
1 Year	44.1%
5 Years	56.7%
10 Years	58.4%
20 Years	59.0%
100 Years	59.1%

Finally, the cost of a protective put option increases with time to expiration. Therefore, because it costs more to insure against losses over longer periods than shorter periods, it follows that risk does not diminish with time.

Risky asset	100
Risk-free rate	3%
Volatility	20%
Strike Price	95

Time to	Price of
Expiration	Put Option
0.25	1.67
1	4.39
5	8.61
10	9.49

## ERROR MAXIMIZATION

Fallacy: Asset allocation determines more than 90 percent of performance

Fallacy: Time diversifies risk

#### Fallacy: Optimized portfolios are hypersensitive to input errors

Fallacy: Factors offer superior diversification and noise reduction

## ERROR MAXIMIZATION

#### If assets are close substitutes

Expected returns:	7.8%	7.6%	7.3%
Standard deviations:	25.3%	25.0%	21.5%
Correlations:	0.75	0.66	0.60

	Correct Weights	Incorrect Weights*	Mis- allocation
France	25.5%	-5.7%	-31.2%
Germany	23.8%	44.9%	21.1%
United Kingdom	50.7%	60.8%	10.1%
Total			62.4%

#### Probability of 10% or greater loss

Difference

End of 1 year	19.2%	20.1%	0.9%
Within 1 year	47.6%	48.8%	1.2%

#### If assets are <u>not</u> close substitutes

Expected returns:	8.8%	4.1%	6.2%
Standard deviations:	16.6%	5.7%	20.6%
Correlations:	0.10	0.16	-0.07

	Correct Weights	Incorrect Weights*	Mis- allocation
U.S. Equities	66.7%	55.0%	-11.7%
U.S. Treasuries	19.8%	27.0%	7.2%
Commodities	13.5%	18.0%	4.5%
Total			23.5%

Probability of 10% or greater loss

#### Difference

End of 1 year	6.1%	4.4%	-1.7%
Within 1 year	17.7%	12.9%	-4.9%

In both examples, we introduce expected return errors as follows: we increase the expected returns of the first asset and the third asset by 1%, and we decrease the expected return of the second asset by 1%.

Source: A Practitioner's Guide to Asset Allocation, Wiley 2017 Analysis is based on data spanning Jan 1976 through Dec 2015. Fallacy: Asset allocation determines more than 90 percent of performance

Fallacy: Time diversifies risk

Fallacy: Optimized portfolios are hypersensitive to input errors

#### Fallacy: Factors offer superior diversification and noise reduction

## FACTORS

- Some investors believe that factors offer greater potential for diversification than asset classes because they appear less correlated than asset classes.
- Factors appear less correlated only because the portfolio of assets designed to mimic them includes short positions.
- Given the same constraints and the same investible universe, it is mathematically impossible to regroup assets into factors and produce a better efficient frontier.

## FACTORS



Asset classes (correlations range from -0.16 to 0.82)

#### Principal components (all factors uncorrelated)

Notes: This analysis incorporates the following asset classes: U.S. large cap, U.S. small cap, EAFE equities, emerging equities, global sovereigns, U.S. government bonds, U.S. corporate bonds, commodities, and hedge funds. It is based on monthly returns over the period Jan 1990 through Dec 2013. Excess returns represent the return over the risk-free rate. All data obtained from DataStream.

## FACTORS

- Some investors believe that consolidating a large group of securities into a few factors reduces noise more effectively than consolidating them into a few asset classes.
- Consolidation reduces noise around means but no more so by using factors than by using asset classes.
- Consolidation does not reduce noise around covariances.



#### Challenge: Identify acceptable portfolios without imposing constraints

Challenge: Determine the optimal exposure to illiquid assets

Challenge: Construct optimal portfolios in the presence of estimation error

Challenge: Construct portfolios to accommodate shifting risk regimes

## CONSTRAINTS

Mean-variance optimization:

$$E(U) = \mu_p - \lambda_{RA} \sigma_p^2$$

Mean-variance-tracking error optimization:

$$E(U) = \mu_p - \lambda_{RA}\sigma_p^2 - \lambda_{TEA}\xi_p^2$$

## CONSTRAINTS

#### Efficient Surface



## CONSTRAINTS

Iso-Expected Return Curve



Standard Deviation

Challenge: Identify acceptable portfolios without imposing constraints

Challenge: Determine the optimal exposure to illiquid assets

#### Challenge: Construct optimal portfolios in the presence of estimation error

Challenge: Construct portfolios to accommodate shifting risk regimes

## LLIQUIDITY

#### Shadow assets and liabilities

The value of liquidity and cost of illiquidity will differ for each investor. This illustration offers plausible estimates.

	Return	Volatility		
Shadow assets (attached to liquid assets)				
Tactical asset allocation	40	80		
Shadow liabilities (attached to illiquid assets)				
Sub-optimality: weight drift	16	0		
Sub-optimality: cash demands	18	0		
Borrowing: cash demands	17	10		

#### Optimal allocation to real estate

The optimal allocation decreases when we account for illiquidity.



Ignoring IlliquidityAccounting for Illiquidity

Challenge: Identify acceptable portfolios without imposing constraints

Challenge: Determine the optimal exposure to illiquid assets

#### Challenge: Construct optimal portfolios in the presence of estimation error

Challenge: Construct portfolios to accommodate shifting risk regimes

- When investors estimate asset class covariances from historical returns they face three types of estimation error: small-sample error, independent-sample error, and interval error.
- Small-sample error arises because the investor's investment horizon is typically shorter than the historical sample from which covariances are estimated.
- Independent-sample error arises because the investor's investment horizon is independent of history.
- Interval error arises because investors estimate covariances from higher frequency returns than the return frequency they care about.
- Common approaches for controlling estimation error, such as Bayesian shrinkage and resampling, make portfolios less sensitive to estimation error.
- A new approach, called stability-adjusted optimization delivers portfolios that rely more on relatively stable covariances and less on relatively unstable covariances.

#### Small-Sample Independent-Sample Interval Mapping Error Error Error Error : ÷ ÷ ÷ : Ħ $\leftrightarrow$ Ħ $\prod$ $\leftrightarrow$ : $\leftrightarrow$ $\leftrightarrow$ : : ÷ ÷ ÷ : Vary: Small samples Vary: Forecasting sample Vary: Measurement interval Vary: Factor mapping Hold constant: Hold constant: Hold constant: Hold constant: Forecasting sample Sample size Sample size Sample size Factor mapping Factor mapping Forecasting sample Forecasting sample Measurement interval Measurement interval Factor mapping Measurement interval

#### Components of Estimation Error

#### Begin with a long sample of asset returns Estimate small sample $\Sigma_{s1}$ $\Sigma_{s2}$ $\Sigma_{s3}$ $\Sigma_{sn}$ ... covariance matrices $\Sigma_{e3}$ $\Sigma_{e1}$ $\Sigma_{e2}$ $\Sigma_{en}$ Compute error matrix for each small sample versus ... $= \Sigma_{s2} - \Sigma_{c2}$ its complementary sample $= \Sigma_{s1} - \Sigma_{c1}$ $= \Sigma_{s3} - \Sigma_{c3}$ $= \Sigma_{sn} - \Sigma_{cn}$ $\Sigma_2$ Σ1 Σ3 Add each error matrix to $\Sigma_n$ ... the baseline covariance $= \Sigma_{e2} + \Sigma_{base}$ $= \Sigma_{e3} + \Sigma_{base}$ $= \Sigma_{e1} + \Sigma_{base}$ $= \Sigma_{en} + \Sigma_{base}$ matrix Draw sample returns from ... normal distributions Combine to form a composite non-normal distribution

#### Constructing the Stability-Adjusted Return Distribution

Improvement in 10% Worst Outcomes: Stability Adjustment versus Error Blind





Challenge: Identify acceptable portfolios without imposing constraints

Challenge: Determine the optimal exposure to illiquid assets

Challenge: Construct optimal portfolios in the presence of estimation error

#### Challenge: Construct portfolios to accommodate shifting risk regimes

$$Turbulence_{t} = \frac{1}{N} (\boldsymbol{x}_{t} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\boldsymbol{x}_{t} - \boldsymbol{\mu})$$

 $x_t$  is a vector of monthly returns across asset classes.

 $\mu$  is a vector of average returns for each asset class over the full 40-year sample.

 $\Sigma^{-1}$  is the inverse of the covariance matrix computed from the 40-year sample.

#### Hidden Markov Model Fit and Conditional Asset Class Performance

Hidden Markov Model Fit: Turbulence	Calm	Moderate	Turbulent
Regime Persistence	92%	75%	67%
Turbulence Average	0.7	1.1	1.7
Turbulence Standard Deviation	0.2	0.3	0.6
Average Annual Asset Return	Calm	Moderate	Turbulent
U.S. Equities	15.0%	13.6%	-27.7%
Foreign Developed Market Equities	15.3%	5.7%	-12.0%
Emerging Market Equities	17.2%	21.7%	-26.0%
Treasury Bonds	5.9%	9.6%	12.3%
U.S. Corporate Bonds	7.5%	10.2%	4.2%
Commodities	7.8%	7.8%	-17.1%
Cash Equivalents	3.9%	5.9%	7.4%
Asset Standard Deviations	Calm	Moderate	Turbulent
U.S. Equities	12.6%	20.2%	19.9%
Foreign Developed Market Equities	14.7%	19.6%	31.0%
Emerging Market Equities	21.3%	30.5%	32.5%
Treasury Bonds	4.2%	6.4%	12.1%
U.S. Corporate Bonds	5.1%	7.8%	16.6%
Commodities	18.1%	21.3%	30.3%
Cash Equivalents	0.8%	1.1%	1.7%

Next Period Probability of Each Regime

$$\begin{bmatrix} P(\varphi_{t+1} = A) \\ P(\varphi_{t+1} = B) \\ P(\varphi_{t+1} = C) \end{bmatrix} =$$

$$\begin{bmatrix} P(\varphi_{t+1} = A | \varphi_t = A) & P(\varphi_{t+1} = A | \varphi_t = B) & P(\varphi_{t+1} = A | \varphi_t = C) \\ P(\varphi_{t+1} = B | \varphi_t = A) & P(\varphi_{t+1} = B | \varphi_t = B) & P(\varphi_{t+1} = B | \varphi_t = C) \\ P(\varphi_{t+1} = C | \varphi_t = A) & P(\varphi_{t+1} = C | \varphi_t = B) & P(\varphi_{t+1} = C | \varphi_t = C) \end{bmatrix} \begin{bmatrix} P(\varphi_t = A) \\ P(\varphi_t = B) \\ P(\varphi_t = C) \end{bmatrix}$$

Estimated probability of each regime occurring next month, calibrated on prior data at each point in time.

Cumulative returns of static portfolios, and

tactical strategy that

allocates proportional

to regime forecasts.



#### Regime forecasts

Backtest performance

